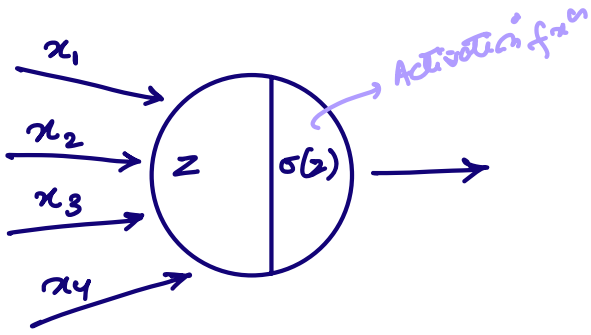
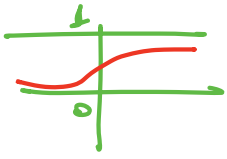
Artificial Neural Network

Neuron

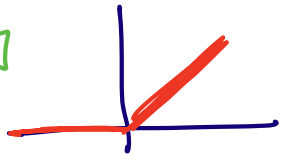


$$z = w_1x_1 + w_2x_2 + w_3x_3 + w_4x_4 + b$$



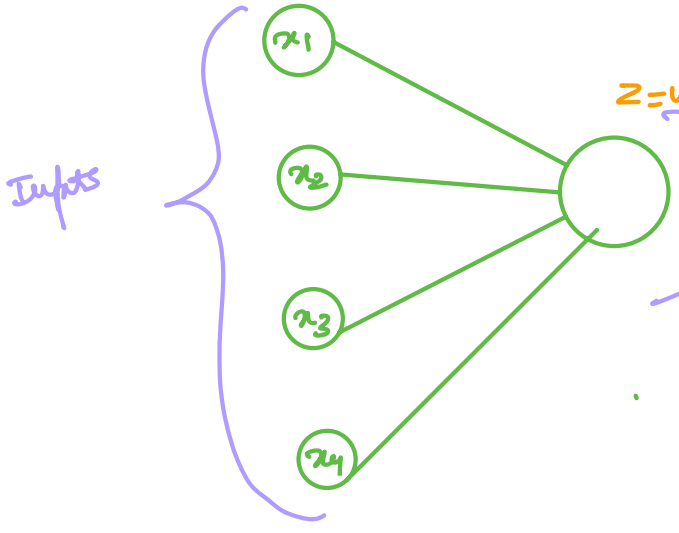
Log. Regression: $\sigma(z) = \frac{1}{1+e^{-z}}$ → Sigmoid → $[0,1]$

Relu → $[0,\infty]$



$\sigma(z) \geq 0.5 \rightarrow 1$
$\sigma(z) < 0.5 \rightarrow 0$

1 layer Architecture



$z = w_1x_1 + w_2x_2 + w_3x_3 + w_4x_4 + b$

$\sigma(z)$
↳ activation fn^{n}
output

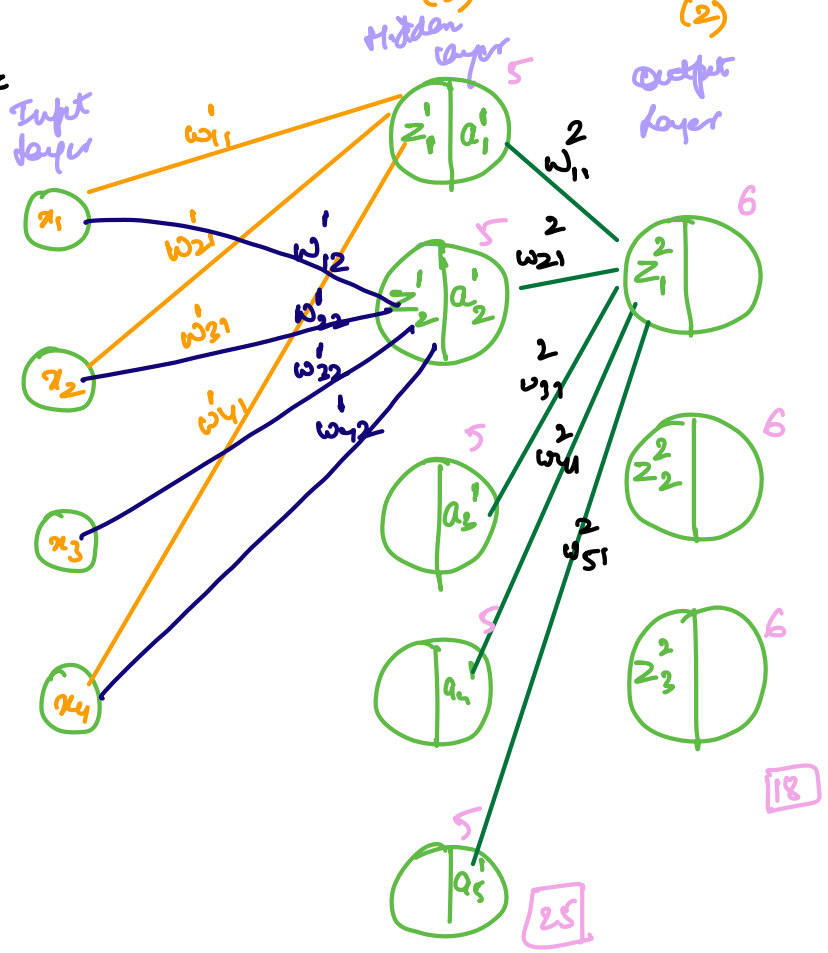
$\sigma(z)$: Sigmoid

$\sigma(z) = \frac{1}{1+e^{-z}}$

$\sigma(z)$: Relu

$\sigma(z) = z \quad z > 0$
$= 0 \quad z \leq 0$

2 layer Architecture



$$z_1^{[1]} = x_1 * w_{11}^1 + x_2 * w_{21}^1 + x_3 * w_{31}^1 + x_4 * w_{41}^1 + b_1^1 = \text{Real no.}$$

$$a_1^{[1]} = \sigma(z_1^{[1]})$$

$$z_2^1 = x_1 * w_{12}^1 + x_2 * w_{22}^1 + x_3 * w_{32}^1 + x_4 * w_{42}^1 + b_2^1 = \text{Real no.}$$

$$a_2^1 = \sigma(z_2^1)$$

$$z_1^2 = a_1^1 * w_{11}^2 + a_2^1 * w_{21}^2 + a_3^1 * w_{31}^2 + a_4^1 * w_{41}^2 + a_5^1 * w_{51}^2 + b_1^2$$

Parameters Learn:

$25 + 18 = 43$

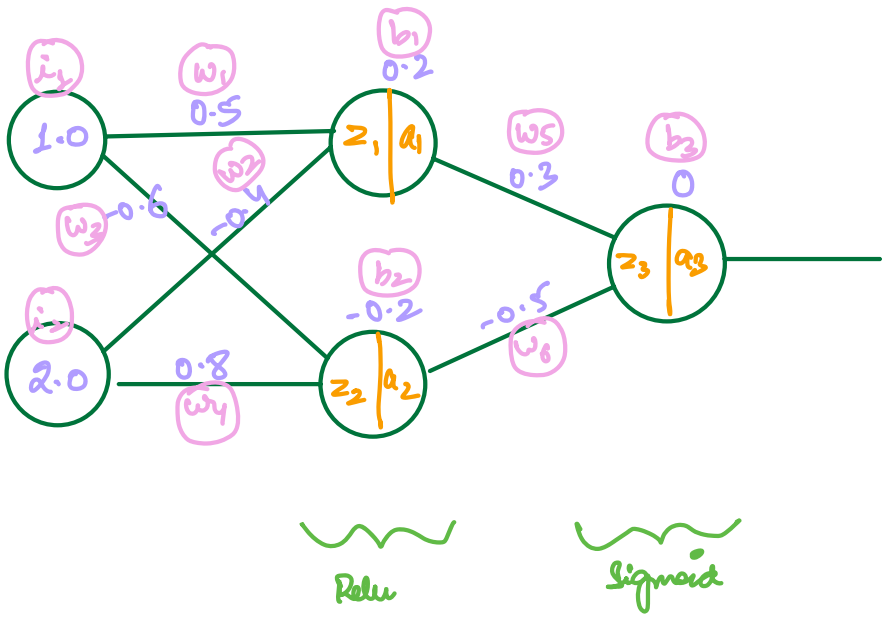
Softmax Activation for [1,2,3]

$$\frac{e^1}{e^1+e^2+e^3}, \quad \frac{e^2}{e^1+e^2+e^3}, \quad \frac{e^3}{e^1+e^2+e^3}$$

4. Answer *any TWO* of the followings

[a] Consider a two-layer neural network used for binary classification. The network has an input layer with 2 neurons, a hidden layer with 2 neurons, and an output layer with 1 neuron. The activation function for the hidden layer is ReLU (Rectified Linear Unit), and for the output layer, it's a sigmoid function. The network is trained using the binary cross-entropy loss function and stochastic gradient descent (SGD) with a learning rate of 0.01. The initial weights and biases are as follows: Weights from input to hidden layer: W_1 = [[0.5, -0.6], [-0.4, 0.8]], Bias for hidden layer: b_1 = [0.2, -0.2], Weights from hidden to output layer: W_2 = [0.3, -0.5], Bias for output layer: b_2 = 0. Consider the network is trained with a single training sample (X = [1.0, 2.0], Y = 0). Perform the forward pass to calculate activations at hidden layer and output layer, and then compute the loss. **[4] [CO2]**

[b] Consider the neural network in 4[a] again and perform the backpropagation to update the weights and biases. Calculate the updated weights W_1, W_2, and biases b_1, b_2 after one iteration. Show your calculations for the forward pass, loss calculation, and backpropagation steps. **[4] [CO2]**



$z_1 = i_1 * w_1 + i_2 * w_2 + b_1 = 1 * 0.5 + 2 * (-0.4) + 0.2 = -0.1$

$a_1 = Relu(z_1) = 0$

$Relu(z_1): \begin{cases} 0 & z_1 \leq 0 \\ z_1 & z > 0 \end{cases}$

$z_2 = i_1 * w_3 + i_2 * w_4 + b_2 = 1 * (-0.6) + 2 * (0.8) - 0.2$

$= 0.8$

$a_2 = Relu(z_2) = 0.8$

$z_3 = a_1 * w_5 + a_2 * w_6 + b_3 = 0 * 0.3 + 0.8 * (-0.5) + 0 = -0.4$

$a_3 = Sigmoid(z_3) = \frac{1}{1 + e^{-z_3}} = \frac{1}{1 + e^{0.4}} = 0.401$